ESTIMATING POPULATION MEAN USING RATIO ESTIMATOR FOR MEAN IS LESS THAN VARIANCE IN SIMPLE RANDOM SAMPLING

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Abstract:
This paper presents a thorough investigation into the estimation of population mean using ratio estimators within the framework of simple random sampling, with a particular focus on scenarios where the population mean is less than the variance. The paper underscores the importance of including auxiliary variables and their specific measures in the construction of ratio estimators. Factors such as sample size, coefficient of variation, kurtosis, and correlation coefficients prove to be valuable in improving estimation accuracy across different sample sizes, particularly when the population mean is less than the variance. The proposed estimator is based on the ratio of the sample mean and the sample median of the variable of interest, and it Incorporates the information on the population size and the sample size. We derive the bias and the mean squared error of the proposed estimator, and compare its performance with some existing ratio estimators using simulated and real data sets. The results show that the proposed estimator has smaller bias and mean squared error than the conventional ratio estimator and some other ratio estimators under certain conditions. We also provide some recommendations for choosing an appropriate ratio estimator depending on the characteristics of the population.
Keywords: *Ratio Estimator, Simple Random Sampling, variance is Less, Population Mean, Mean square Error, Relative efficiency*

INTRODUCTION

Simple Random Sampling

Simple Random Sampling is a commonly employed statistical sampling method. In this technique, each element or individual within the population has an equal and independent probability of being chosen for inclusion in the sample. This approach serves as the basis for a range of estimation methods, one of which is the ratio estimator. Utilizing a ratio estimator for estimating the population mean in simple random sampling is a valuable statistical approach. It is often expected that the ratio estimator will yield an estimate that is less than the population variance, given the inherent properties of data dispersion, the influence of sample variation, and potential bias correction. Understanding this relationship is essential for accurate estimation and inference in statistics.

Ratio Estimation

Estimating the population mean of an important variable with the use of contextual information is a common application of ratio estimation. The ratio estimator in simple random sampling is the ratio of the means of the sample for the dependent and independent variables. It is well-known that the ratio estimator is biased, and many strategies have been developed to address this. Most of these strategies, however, count on the fact that the variance of the characteristic of interest in the population is larger than or equal to the mean of the population, which is not always the case. For example, if the variable of interest is a count or a proportion, it is possible that the population mean is greater than the population variance. In such cases, the conventional ratio estimator and its corrections may not perform well and may lead to large errors.

Regression Estimator

When the dependent and independent variables in a study are correlated (whether positively or negatively), the results can be analyzed using the Regression Estimator. An alternative to the ratio or product estimator is the regression estimator if the relationship does not go via the origin. Regression estimator is presented as

$\mu_{yl} = \bar{y} + b_1(\mu_x - \bar{x})$,

More Use of Auxiliary Variable

Auxiliary variable also used in the ratio and regression estimator in term of coefficient of skewness, coefficient of kurtosis and quantiles. Some details of these measures are presented below.

Skewness

When a set of data doesn't follow the symmetrical bell curve, or normal distribution, we say that it has skewness. A skewed curve is one that has been tilted to the left or right. The degree to which a distribution deviates from normality is a measure of its skewness. For example, a lognormal distribution would be right-skewed while the skew of a normal distribution is zero.

The degree of skewness can be evaluated in various ways. Common skewness measures include Pearson's scond and scond coefficients. Person mode skewness, or Pearson's first coefficient of skewness, is calculated by taking the difference between the mode and the mean and dividing it by the standard deviation. To calculate the Pearson median skewness, also known as the second coefficient of skewness by Person, we first subtract the median from the mean, then multiply that number by three, and finally divide the resulting number by the standard deviation.

Pearson's skewness formulas are provided.

$$Sk_1 = \frac{\bar{X} - M_0}{s}$$

$$Sk_2 = 3\frac{\bar{X} - M_d}{s}$$

Kurtosis

In simple random sampling, if you find that the kurtosis (using a ratio estimator for the mean) is less than the variance, it suggests that the distribution may have lighter tails compared to a normal distribution. This means there are fewer extreme values in your sample data compared to what you would expect in a normal distribution. However, this observation doesn't necessarily indicate a problem; it characterizes the shape of the distribution within the context of your sample. It's crucial to choose appropriate sampling methods and estimators based on your data and research objectives.

The kurtosis of a distribution is the degree to which its tails deviate from the normal distribution's tails. Simply said, kurtosis measures the presence or absence of outlying values in the tails of a distribution. Kurtosis is a statistical measure of dispersion used to assess financial risk.

Quartile deviation

The quartile is a measure of the range of values from the top and bottom of the distribution. The quartile system creates four groupings of data by dividing it into three points (the lowest, middle, and highest values).

Half the gap between the top and bottom quartiles multiplied by itself is the quartile deviation. We can define mathematically as:

Quartile deviation $= \frac{(Q_3 - Q_1)}{2}$.

You can use the quartile deviation to get a sense of the range in where the middle 50% of your sample data falls, which is useful for analysing the dispersion of a distribution around a measure of its central tendency, such as the mean or average.

When a data collection is divided into quartiles, like the median divides it in half, we can learn about the distribution of the data. The quartiles are determined by adding together the scores on either side of the cutoff and dividing by two.

Twenty-five percent (25%) of the distribution is the first quartile, also known as the lowest quartile. The score points known as quartiles are used to divide a distribution into four equal groups.

Relative efficiency

Efficiency is a metric used to evaluate the relative merits of different statistical methods for estimating, designing experiments, and evaluating hypotheses. One can attain the same level of performance with fewer observations when using a more efficient estimator, experiment, or test.

The ratio of the MSEs of two estimators is a good measure of their relative accuracy. This is the same as the variance ratio if the two estimators are independent.

Comparing a particular technique to the notational "best possible" typically necessitates considering the relative efficiency of both procedures. It is possible to utilise an asymptotic relative efficiency, which is defined as the limit of the relative efficiencies as the sample size increases, but in practise, the efficiencies and relative efficiencies of two methods depend on the sample size available for the particular procedure. An efficient estimator has a small variance or MSE, meaning that the difference between the true value and the predicted value is negligible. If both estimators, E1 and E2, are unbiased, then the one with the smaller variance is often used. Effectiveness of E1 compared to E2

RE $[\frac{E_1}{E_2}]=\frac{V(E_2)}{V(E_1)}$

Tailor and Sharma (2009) presented the ratio-cum-product approximation of the mean of a finite population by utilising the coefficient of variation and the information coefficient of kurtosis. The sample mean estimator, the typical ratio estimator, and the product estimator have all been found to be less effective than the proposed estimator. It was also shown that the proposed estimators have merit. The effectiveness of the estimator was enhanced by using supplementary data during both the estimating and selection phases. The positive association between the auxiliary varying and the study variate indicated the need for an estimating step as well as a selection stage to boost the effectiveness of estimators. The mean of the population was calculated using the ratio technique.

Naeem Shahzad and Abida (2022) In a framework of simple random sampling, this paper explores the use of ratio estimators to determine the population mean of a study variable when the mean is larger than the variance. The authors evaluate and suggest 52 new ratio estimators using a wide range of methods and techniques. These estimators, together with their respective mean square errors, are presented in the publication. When the variation is larger than the mean, a simulation study is used to make the comparison easier. Finding the most accurate estimates of the population parameter of interest is the goal. Several estimators are discussed that use only one auxiliary variable. According to their research, the $\widehat{\mathrm{m}}_{12}$ estimator is the second most efficient for any sample size, and the $\widehat{\mathrm{m}}_{36}$ estimator is the third most efficient. $\widehat{\mathrm{m}}_{17}$, which ranks fifth, is a strong performer with a sample size of 30. It is determined that the most efficient estimator for samples of 40, 50, 60, 100, and 200 is $\widehat{\mathrm{m}}_{38}$. Both the correlation coefficient and the regression-cum ratio are incorporated into this estimator. Finally, the paper finishes with some guidelines for picking a reliable ratio estimator that takes into account the specifics of the target population.

Yan and Tian (2010) proposed ratio type estimator for population mean by using known skewness coefficient of auxiliary variable ratio estimator was obtained by using expression of MSE up to the 1^{ST} order of approximation. The efficiency between some known estimators was compared with proposed estimator and conclusion was supported with original data by using an application.

Naeem Shahzad et al., (2021). This paper discusses the use of a ratio estimator with a simple random sample to get the population mean of the research variable. The authors attempted to make a comprehensive comparison of the available ratio estimators and proposed around forty-two ratio estimators that vary in application throughout time and methodology. The mean square errors of each estimator are provided. They employ a simulation study with varying cases of mean equal to variance to draw comparisons. Finding reliable estimators of a parameter in an unknown population is the crux of the problem. Several estimators that rely on a single auxiliary variable are discussed. The fourth efficient estimator, $\widehat{\mathrm{m}}_{16}$, combines the traditional ratio estimator with the simple mean and works well with big samples. For a sample size of 30, m15 ranks fifth. The most effective estimator is found to be $\widehat{\mathrm{m}}_{38}$ for samples of 40, 50, and 60. The correlation coefficient and the regression-cum ratio estimator are two methods that can be used to generate these estimates.

Enang, E. et al., (2014). the paper proposes a new ratio estimator based on quartile deviation, kurtosis coefficient, and tri-mean as auxiliary variables. It derives the bias and MSE of the proposed estimator and compares it with some existing estimators using numerical examples. It also suggests some conditions for choosing an appropriate auxiliary variable.

Singh (2011) extensively discussed the use of auxiliary information for enhancing estimators in sample surveys. In this light, the product and ratio methods of estimate stand out as particularly applicable. When the auxiliary and study variables had a strong positive or negative association, it was especially challenging to make improvements using these methods. By many authors considerable attempts were done in recent years to improve depth of the product methods of estimation and conventional ratio by using proper transformation on auxiliary variable X with the help of some known parameters of the auxiliary variable cx coefficient of variation,σx (S.D) B_2(x) coefficient of kurtosis,ρ correlation coefficient between the study variable y and the auxiliary variable x.

Subramani all and Kumarapadiya (2012) proposed a method for estimating a population median and kurtosis coefficient using a linear combination of data from the sample population. They evaluated the proposed estimator to previously established modified ratio estimators and obtained its bias and MSE. They also determined the parameters under which the proposed estimators outperform the standard modified ratio estimators.

Tailor, Tailor,Parma and Kumar (2012) proposed that it was significant to use the auxiliary information for the improvement of efficiency of estimators of unknown population parameters. The dual to ratio-cum-product estimator for population mean was used to know the parameters of auxiliary variable. In this paper bias and MSE expression were also obtained at high degree of approximation. Also compared empirically and theatrically the suggested parameters estimators. It was also proposed that dual to ratio and product estimators to find out population mean.

While estimating the population mean of the research variable, Yadav and Kadilar (2013) considered several estimators and suggested a generic family of estimators that relied on the known value of particular populations' parameters on the auxiliary variable. In this paper, the authors introduced a new class of estimators for the mean of a population. The current family of estimators is a big step up from what came before. Numerical evidence also supported this conclusion.

Lu and yan (2013) suggested that the efficiency of the estimators may be improved through the use of auxiliary information. The MSE equation for the proposed estimator class was calculated using two auxiliary variables. When estimating with a positive ratio, they made use of the relationship between the auxiliary and research variables. When a negative correlation was found, the product method of estimation was chosen. There was some thought given to ratio estimators with two auxiliary variables based on a finite number mean for the variable of interest.

When the auxiliary variable was known, Subramani (2013) investigated the revised study variable for estimate using the modified ratio estimator. For a set population, we calculated the mean squared error (MSE) of the suggested estimator and the biassed version, and we compared them to the standard modified ratio estimator.To add, we derived the conditions under which the suggested estimator outperformed the existing modified ratio approximation for a certain known population. That was demonstrated mathematically.

Yavav, Mishra (2014) and Shukla suggested a linear combination of two efficient estimators of the population mean as a median auxiliary variable. The proposed estimator of the population mean of the research variable was compared to those previously mentioned in the literature. The efficiency of estimation methods such the mean, variance, kurtosis, variation, skewness median, etc., was enhanced by the usage of these parameters.

Subramani and Prabavathy (2014) proposes a method for estimating the mean of a finite population by using a median-based modified ratio estimator. For several natural populations, we calculated the mean squared error (MSE) and bias of the suggested estimators as well as contrasting them to those of the ratio estimator, the regression model estimator, the median base ratio estimator, and the sample mean. Comparisons to other estimators, such as those based on linear regression, showed that the recommended median base modified ratio estimator outperformed them all.

Kosgey, Sheryl Chebet, and Leo Odongo (2022) proposes knowledge on the minimum and maximum values of auxiliary data, the article discusses some successful estimators for determining the mean of a limited population. This article uses simulated data sets to compare the suggested estimators to various already known estimators and calculates their bias and mean squared error. The results reveal that under specific conditions, some of the proposed estimators perform better than others in terms of bias and mean squared error.
Cem Kadilar and Hulya Cingi (2004) proposes some new ratio estimators that use the information on the coefficient of kurtosis and the coefficient of variation of the auxiliary variable. The paper shows that these estimators are more efficient than the classical ratio estimator and some other existing estimators. The paper also provides some numerical examples to illustrate the results.
Evrim Oral and Cem Kadilar (2011) introduces some robust ratio-type estimators that use modified maximum likelihood estimators (MMLEs) to deal with non-normality and outliers in the data. The article derives the bias and mean-squared error (MSE) of these estimators and compares them with other Kadilar-Cingi estimators. The article also conducts some simulations under various super-population models to demonstrate the robustness properties of these estimators.
Subzar A et al., (2018) proposes some new ratio estimators that use the information on the population proportion possessing a certain attribute. The paper derives the bias and MSE of these estimators and compares them with other ratio estimators. The paper also gives some numerical examples using empirical data to support the findings.
Cem Kadilar, Hulya Cingi, and N. Balakrishnan (2008). The article adapts the ratio estimation using ranked set sampling, a technique that improves the efficiency of sampling by ranking the units before selecting them. The article uses a ratio estimator based on Prasad (1989) in simple random sampling and extends it to ranked set sampling. The article derives the bias and MSE of this estimator and compares it with other ratio estimators using ranked set sampling. The article also provides some numerical results to show the advantages of this estimator.
For the mean of a study variable y in a finite population, Enang, Akpan, and Ekpenyong (2014) proposed an alternative ratio estimator using data on the mean of an auxiliary variable X that was highly correlated with Y. They then drew conclusions about the bias and MSE of the proposed estimators. Both the numerical and analytical analyses demonstrated that some of the already used estimators were inefficient compared to the proposed alternative estimators. The small differences between the estimator and the alternative estimators for all populations examined suggested that the estimator was superior to the alternatives as a regression estimator.
Ekpenyong et al., (2015) proposed that a ratio estimator that takes into account the departure from the quartile, the kurtosis coefficient, a non-standard measure (the Tri-mean), and the sample size. The research uses simulated data sets to evaluate the proposed estimator to some already known ratio estimators and determines its bias and mean square error. In some cases, the suggested estimator outperforms other ratio estimators in terms of bias and mean square error. In order to improve the estimators' performance.
Lu and Yan (2014) proposed using the auxiliary data. For the purpose of estimating the population mean and the research variable, this data was often input into a regression, product, or ratio type estimator. The positive ratio technique of estimation was employed to determine the degree of association between the auxiliary variable and the research variable. When the correlation was -ve, however, the product method of estimation was chosen. Regression type estimators and ratio product estimators have both been studied with the use of the auxiliary variable.

METHODOLOGY

In this section, we will examine estimators for the population mean that are being studied for comparison purposes. These estimators utilize a single auxiliary variable in the context of simple random sampling. we will provide the mean square error associated with each estimator.
Cochran's (1940) standard ratio estimator for use with a random sample is presented below.

$$\hat{m}_1 = \frac{\bar{y}}{\bar{x}}\bar{X}$$

For the above estimator the mean square error is presented below

$$MES(\widehat{m}_1) \cong \frac{1-f}{n}(R^2 S_X^2 - 2RS_{XY} + S_Y^2), \quad \text{Where}, R = \frac{\bar{Y}}{\bar{X}}$$

Kadilar and Cingi (2004) introduced a population mean estimator, drawing upon the approach outlined by Ray and Singh (1981).The estimator is expressed below

$$\widehat{m}_6 = \frac{\bar{y} + b_{yx}(\bar{X} - \bar{x})}{\bar{x}}\bar{X}$$

For the above estimator the mean square error given below

$$MSE(\widehat{m}_6) \cong \frac{(1-f)}{n}[R^2 S_X^2 + S_Y^2(1-\rho^2)], \quad \text{where } b_{yx} = \frac{S_{xy}}{S_x^2}$$

Swain (2014) introduced an alternative exponential estimator with a ratio-type approach. characterized by its unique ratio-type methodology. This innovative estimator represents a departure from conventional estimation techniques and was designed to offer an alternative and potentially improved approach to solving specific statistical or data analysis problems.

$$\widehat{m}_{33} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{1/2}$$

The MSE

$$\text{MSE}(\widehat{m}_{33}) = \frac{1-f}{n}\bar{Y}^2\left[c_y^2 + \frac{C_x^2}{4} - \rho C_y C_x\right].$$

Abdullahi and Yahaya (2017) introduced a new estimator for a specific statistical problem. They were influenced by the pioneering work of Housila and Neha-Agnihotri (2008) and built upon their insights to create an original approach.

$$\widehat{m}_{43} = \frac{\bar{y}}{2}\left[\left(1 + \frac{\widehat{K}}{\theta}\right)\left(\frac{\bar{X}C_x + M_d}{\bar{x}C_x + M_d}\right) + \left(1 + \frac{\widehat{K}}{\theta}\right)\left(\frac{\bar{x}C_x + M_d}{\bar{X}C_x + M_d}\right)\right]$$

For the above estimator the mean square error presented below

$$\text{MSE}(\widehat{m}_{43}) = \frac{(1-f)}{n}S_y^2(1-\rho^2)$$

Noor-ul-Amin (2019)

$$\widehat{m}_{52} = \frac{Z_y}{Z_x}\bar{X}$$

MSE $(\widehat{m}_{52}) = \frac{1-f}{n}\frac{\lambda}{2-\lambda}\left[R^2 S_x^2 - 2RS_{xy} + S_y^2\right]$

where

$$Z_Y = \lambda\bar{y} + (1-\lambda)Z_{y-1}$$
$$Z_X = \lambda\bar{x} + (1-\lambda)Z_{x-1}$$

$\mu_x = 10,\ \mu_x = 8,\ \sigma_x^2 = 2,\ \sigma_y^2 = 3,\ \sigma_{xy} = 1\ ,\ \rho_{xy} = 0.50$

FINDINGS AND DISCUSSIONS

Simulation Study

The research rigorously evaluates these estimators through a simulation study, drawing 50,000 samples of varying sizes from simple random sampling schemes. This empirical approach provides valuable insights into estimator performance under real-world conditions, enhancing the credibility of the findings.

We conducted a simulation study to compare the performance of different ratio estimators in terms of mean squared error (MSE) and relative efficiency (RE). We used simple random sampling to select 50,000 samples of various sizes (n=30,40,50,60,100 and 200) from sampling schemes of size 5000 with specified parameters. We calculated the MSE of each estimator using the formula

$$MSE(\hat{\theta}) = \frac{1}{50,000}\sum_{i=1}^{50,000}(\hat{\theta}_i - \theta)^2$$

where $\hat{\theta}$ is the ratio estimator and θ is the true value of the parameter. We also computed the RE of each estimator relative to the sample mean estimator using the formula

$$RE(\hat{\theta}, \bar{y}) = \frac{MSE(\bar{y})}{MSE(\hat{\theta})}$$

where $\bar{y}$ is the sample mean estimator and the results of the simulation study are presented in Tables 1 for MSE values and Tables 2 and 3 for RE values.

Table 1: MSE for sample size 30,40,50,60,100 and 200

Estimators	n=30	n=40	n=50	n=60	n=100	n=200
$\hat{m}_{.1}$	0.42321	0.30605	0.23792	0.19587	0.10907	0.04780
$\hat{m}_2$	0.36004	0.26443	0.20757	0.17157	0.09667	0.04274
$\hat{m}_3$	0.34143	0.25179	0.19821	0.16392	0.09271	0.04110
$\hat{m}_4$	0.39136	0.28533	0.22292	0.18393	0.10303	0.04534
$\hat{m}_5$	0.35596	0.26168	0.20554	0.16991	0.09582	0.04238
$\hat{m}_6$	0.45615	0.32919	0.25550	0.21046	0.11697	0.05117
$\hat{m}_7$	0.38033	0.27908	0.21888	0.18114	0.10193	0.04500
$\hat{m}_8$	0.35694	0.26309	0.20695	0.17141	0.09683	0.04287
$\hat{m}_9$	0.41833	0.30454	0.23764	0.19626	0.10975	0.04822
$\hat{m}_{10}$	0.37528	0.27565	0.21633	0.17907	0.10085	0.04455
$\hat{m}_{11}$	29.57038	16.15186	9.70555	7.18210	2.26706	0.46522
$\hat{m}_{12}$	0.10580	0.07651	0.05948	0.04897	0.02727	0.01195
$\hat{m}_{13}$	0.90453	0.60900	0.45382	0.36502	0.19389	0.08215
$\hat{m}_{14}$	0.37142	0.27208	0.21321	0.17613	0.09903	0.04371
$\hat{m}_{15}$	0.41964	0.30376	0.23627	0.19456	0.10841	0.04753
$\hat{m}_{16}$	0.31354	0.23251	0.18408	0.15192	0.08652	0.03861
$\hat{m}_{17}$	0.30578	0.22744	0.18039	0.14942	0.08552	0.03829
$\hat{m}_{18}$	0.42366	0.30635	0.23813	0.19603	0.10915	0.04783
$\hat{m}_{19}$	0.31347	0.23266	0.18438	0.15213	0.08673	0.03874
$\hat{m}_{20}$	0.42337	0.30616	0.23799	0.19592	0.10910	0.04781
$\hat{m}_{21}$	0.42350	0.30624	0.23805	0.19597	0.10912	0.04782
$\hat{m}_{22}$	0.35942	0.26401	0.20726	0.17132	0.09654	0.04268
$\hat{m}_{23}$	0.32331	0.23935	0.18898	0.15627	0.08873	0.03947
$\hat{m}_{24}$	0.33198	0.24532	0.19341	0.15996	0.09065	0.04025
$\hat{m}_{25}$	0.34473	0.25405	0.19989	0.16530	0.09342	0.04140
$\hat{m}_{26}$	0.34459	0.25454	0.20053	0.16613	0.09405	0.04171
$\hat{m}_{27}$	0.36117	0.26600	0.20913	0.17320	0.09777	0.04326
$\hat{m}_{28}$	0.32932	0.24349	0.19205	0.15883	0.09006	0.04001
$\hat{m}_{29}$	0.34854	0.25664	0.20181	0.16687	0.09424	0.04173
$\hat{m}_{30}$	0.33292	0.24596	0.19389	0.16036	0.09085	0.04034
$\hat{m}_{31}$	0.31342	0.23263	0.18435	0.15211	0.08671	0.03873
$\hat{m}_{32}$	0.31351	0.23270	0.18441	0.15215	0.08674	0.03874
$\hat{m}_{33}$	0.33560	0.24729	0.19464	0.16079	0.09094	0.04033

$\hat{m}_{34}$	0.31269	0.23199	0.18376	0.15173	0.08653	0.03867
$\hat{m}_{35}$	0.45161	0.32445	0.25031	0.20499	0.11239	0.04826
$\hat{m}_{36}$	0.13247	0.09399	0.07236	0.05887	0.03316	0.01452
$\hat{m}_{37}$	0.13253	0.09402	0.07237	0.05888	0.03317	0.01453
$\hat{m}_{38}$	0.37969	0.27658	0.21664	0.17638	0.09922	0.04406
$\hat{m}_{39}$	0.54133	0.38008	0.28846	0.23385	0.12492	0.05208
$\hat{m}_{40}$	0.38482	0.28213	0.22113	0.18298	0.10289	0.04539
$\hat{m}_{41}$	0.32297	0.23939	0.18913	0.15661	0.08900	0.03959
$\hat{m}_{42}$	0.41098	0.29968	0.23407	0.19341	0.10828	0.04762
$\hat{m}_{43}$	0.31493	0.23315	0.18442	0.15209	0.08650	0.03856
$\hat{m}_{44}$	0.31581	0.23363	0.18471	0.15229	0.08657	0.03857
$\hat{m}_{45}$	0.51831	0.38322	0.30292	0.24539	0.14054	0.05903
$\hat{m}_{46}$	0.45193	0.32648	0.25355	0.20892	0.11619	0.05085
$\hat{m}_{47}$	0.45345	0.32746	0.25426	0.20947	0.11647	0.05096
$\hat{m}_{48}$	0.31367	0.23278	0.18419	0.15236	0.08674	0.03866
$\hat{m}_{49}$	0.45463	0.32822	0.25480	0.20990	0.11669	0.05105
$\hat{m}_{50}$	0.31232	0.23182	0.18348	0.15173	0.08641	0.03852
$\hat{m}_{51}$	0.38482	0.28213	0.22113	0.18298	0.10289	0.04539
$\hat{m}_{52}$	0.01006	0.00733	0.00573	0.00464	0.00285	0.00116

Table 2: RE for sample size 30,40,50,60,100 and 200

Estimators	n=30	n=40	n=50	n=60	n=100	n=200
$\hat{m}_{1}$	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
$\hat{m}_{2}$	1.17545	1.15740	1.14622	1.14163	1.12827	1.11839
$\hat{m}_{3}$	1.23952	1.21550	1.20034	1.19491	1.17646	1.16302
$\hat{m}_{4}$	1.08138	1.07262	1.06729	1.06492	1.05862	1.05426
$\hat{m}_{5}$	1.18893	1.16956	1.15754	1.15279	1.13828	1.12789
$\hat{m}_{6}$	0.92779	0.92971	0.93119	0.93068	0.93246	0.93414
$\hat{m}_{7}$	1.11274	1.09664	1.08699	1.08132	1.07005	1.06222
$\hat{m}_{8}$	1.18566	1.16329	1.14965	1.14270	1.12641	1.11500
$\hat{m}_{9}$	1.01167	1.00496	1.00118	0.99801	0.99380	0.99129
$\hat{m}_{10}$	1.12772	1.11029	1.09980	1.09382	1.08151	1.07295
$\hat{m}_{11}$	0.01431	0.01895	0.02451	0.02727	0.04811	0.10275
$\hat{m}_{12}$	4.00010	4.00013	4.00000	3.99980	3.99963	4.00000
$\hat{m}_{13}$	0.46788	0.50255	0.52426	0.53660	0.56254	0.58186
$\hat{m}_{14}$	1.13944	1.12485	1.11590	1.11208	1.10138	1.09357

$\widehat{m}_{15}$	1.00851	1.00754	1.00698	1.00673	1.00609	1.00568
$\widehat{m}_{16}$	1.34978	1.31629	1.29248	1.28930	1.26063	1.23802
$\widehat{m}_{17}$	1.38403	1.34563	1.31892	1.31087	1.27537	1.24837
$\widehat{m}_{18}$	0.99894	0.99902	0.99912	0.99918	0.99927	0.99937
$\widehat{m}_{19}$	1.35008	1.31544	1.29038	1.28752	1.25758	1.23387
$\widehat{m}_{20}$	0.99962	0.99964	0.99971	0.99974	0.99973	0.99979
$\widehat{m}_{21}$	0.99932	0.99938	0.99945	0.99949	0.99954	0.99958
$\widehat{m}_{22}$	1.17748	1.15924	1.14793	1.14330	1.12979	1.11996
$\widehat{m}_{23}$	1.30899	1.27867	1.25897	1.25341	1.22924	1.21105
$\widehat{m}_{24}$	1.27481	1.24755	1.23013	1.22449	1.20320	1.18758
$\widehat{m}_{25}$	1.22766	1.20468	1.19026	1.18494	1.16752	1.15459
$\widehat{m}_{26}$	1.22816	1.20237	1.18646	1.17902	1.15970	1.14601
$\widehat{m}_{27}$	1.17178	1.15056	1.13767	1.13089	1.11558	1.10495
$\widehat{m}_{28}$	1.28510	1.25693	1.23884	1.23321	1.21108	1.19470
$\widehat{m}_{29}$	1.21424	1.19253	1.17893	1.17379	1.15736	1.14546
$\widehat{m}_{30}$	1.27121	1.24431	1.22709	1.22144	1.20055	1.18493
$\widehat{m}_{31}$	1.35030	1.31561	1.29059	1.28769	1.25787	1.23419
$\widehat{m}_{32}$	1.34991	1.31521	1.29017	1.28735	1.25744	1.23387
$\widehat{m}_{33}$	1.26106	1.23762	1.22236	1.21817	1.19936	1.18522
$\widehat{m}_{34}$	1.35345	1.31924	1.29473	1.29091	1.26049	1.23610
$\widehat{m}_{35}$	0.93711	0.94329	0.95050	0.95551	0.97046	0.99047
$\widehat{m}_{36}$	3.19476	3.25620	3.28800	3.32716	3.28920	3.29201
$\widehat{m}_{37}$	3.19332	3.25516	3.28755	3.32660	3.28821	3.28975
$\widehat{m}_{38}$	1.11462	1.10655	1.09823	1.11050	1.09927	1.08488
$\widehat{m}_{39}$	0.78180	0.80523	0.82479	0.83759	0.87312	0.91782
$\widehat{m}_{40}$	1.09976	1.08478	1.07593	1.07045	1.06006	1.05310
$\widehat{m}_{41}$	1.31037	1.27846	1.25797	1.25069	1.22551	1.20738
$\widehat{m}_{42}$	1.02976	1.02126	1.01645	1.01272	1.00730	1.00378
$\widehat{m}_{43}$	1.34382	1.31267	1.29010	1.28786	1.26093	1.23963
$\widehat{m}_{44}$	1.34008	1.30998	1.28807	1.28617	1.25991	1.23931
$\widehat{m}_{45}$	0.81652	0.79863	0.78542	0.79820	0.77608	0.80976
$\widehat{m}_{46}$	0.93645	0.93742	0.93836	0.93754	0.93872	0.94002
$\widehat{m}_{47}$	0.93331	0.93462	0.93574	0.93507	0.93646	0.93799

$\hat{m}_{48}$	1.34922	1.31476	1.29171	1.28557	1.25744	1.23642
$\hat{m}_{49}$	0.93089	0.93245	0.93375	0.93316	0.93470	0.93634
$\hat{m}_{50}$	1.35505	1.32021	1.29671	1.29091	1.26224	1.24091
$\hat{m}_{51}$	1.09976	1.08478	1.07593	1.07045	1.06006	1.05310
$\hat{m}_{52}$	42.06859	41.75307	41.52182	42.21336	38.27018	41.20690

Table 3: Relative Position of Ratio Estimators

Estimators Ranking	n=30	n=40	n=50	n=60	n=100	n=200
1	$\hat{m}_{52}$	$\hat{m}_{52}$	$\hat{m}_{52}$	$\hat{m}_{52}$	$\hat{m}_{52}$	$\hat{m}_{52}$
2	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$	$\hat{m}_{12}$
3	$\hat{m}_{36}$	$\hat{m}_{36}$	$\hat{m}_{36}$	$\hat{m}_{36}$	$\hat{m}_{36}$	$\hat{m}_{36}$
4	$\hat{m}_{37}$	$\hat{m}_{37}$	$\hat{m}_{37}$	$\hat{m}_{37}$	$\hat{m}_{37}$	$\hat{m}_{37}$
5	$\hat{m}_{17}$	$\hat{m}_{17}$	$\hat{m}_{17}$	$\hat{m}_{17}$	$\hat{m}_{17}$	$\hat{m}_{17}$
6	$\hat{m}_{50}$	$\hat{m}_{50}$	$\hat{m}_{50}$	$\hat{m}_{50}$	$\hat{m}_{50}$	$\hat{m}_{50}$
7	$\hat{m}_{34}$	$\hat{m}_{34}$	$\hat{m}_{34}$	$\hat{m}_{50}$	$\hat{m}_{43}$	$\hat{m}_{43}$
8	$\hat{m}_{31}$	$\hat{m}_{16}$	$\hat{m}_{16}$	$\hat{m}_{16}$	$\hat{m}_{16}$	$\hat{m}_{44}$
9	$\hat{m}_{19}$	$\hat{m}_{31}$	$\hat{m}_{48}$	$\hat{m}_{43}$	$\hat{m}_{34}$	$\hat{m}_{16}$
10	$\hat{m}_{32}$	$\hat{m}_{19}$	$\hat{m}_{31}$	$\hat{m}_{31}$	$\hat{m}_{44}$	$\hat{m}_{48}$
11	$\hat{m}_{16}$	$\hat{m}_{32}$	$\hat{m}_{19}$	$\hat{m}_{19}$	$\hat{m}_{31}$	$\hat{m}_{34}$
12	$\hat{m}_{48}$	$\hat{m}_{48}$	$\hat{m}_{32}$	$\hat{m}_{32}$	$\hat{m}_{19}$	$\hat{m}_{31}$
13	$\hat{m}_{43}$	$\hat{m}_{43}$	$\hat{m}_{43}$	$\hat{m}_{44}$	$\hat{m}_{32}$	$\hat{m}_{19}$
14	$\hat{m}_{44}$	$\hat{m}_{44}$	$\hat{m}_{44}$	$\hat{m}_{48}$	$\hat{m}_{48}$	$\hat{m}_{32}$
15	$\hat{m}_{41}$	$\hat{m}_{23}$	$\hat{m}_{23}$	$\hat{m}_{23}$	$\hat{m}_{23}$	$\hat{m}_{23}$
16	$\hat{m}_{23}$	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$	$\hat{m}_{41}$
17	$\hat{m}_{28}$	$\hat{m}_{28}$	$\hat{m}_{28}$	$\hat{m}_{28}$	$\hat{m}_{28}$	$\hat{m}_{28}$
18	$\hat{m}_{24}$	$\hat{m}_{24}$	$\hat{m}_{24}$	$\hat{m}_{24}$	$\hat{m}_{24}$	$\hat{m}_{24}$
19	$\hat{m}_{30}$	$\hat{m}_{30}$	$\hat{m}_{30}$	$\hat{m}_{30}$	$\hat{m}_{30}$	$\hat{m}_{33}$
20	$\hat{m}_{33}$	$\hat{m}_{33}$	$\hat{m}_{33}$	$\hat{m}_{33}$	$\hat{m}_{33}$	$\hat{m}_{30}$
21	$\hat{m}_{3}$	$\hat{m}_{3}$	$\hat{m}_{3}$	$\hat{m}_{3}$	$\hat{m}_{3}$	$\hat{m}_{3}$
22	$\hat{m}_{26}$	$\hat{m}_{25}$	$\hat{m}_{25}$	$\hat{m}_{25}$	$\hat{m}_{25}$	$\hat{m}_{25}$
23	$\hat{m}_{25}$	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{26}$	$\hat{m}_{26}$
24	$\hat{m}_{29}$	$\hat{m}_{29}$	$\hat{m}_{29}$	$\hat{m}_{29}$	$\hat{m}_{29}$	$\hat{m}_{29}$
25	$\hat{m}_{5}$	$\hat{m}_{5}$	$\hat{m}_{5}$	$\hat{m}_{5}$	$\hat{m}_{5}$	$\hat{m}_{5}$
26	$\hat{m}_{8}$	$\hat{m}_{8}$	$\hat{m}_{8}$	$\hat{m}_{22}$	$\hat{m}_{22}$	$\hat{m}_{22}$
27	$\hat{m}_{22}$	$\hat{m}_{22}$	$\hat{m}_{22}$	$\hat{m}_{8}$	$\hat{m}_{2}$	$\hat{m}_{2}$
28	$\hat{m}_{2}$	$\hat{m}_{2}$	$\hat{m}_{2}$	$\hat{m}_{2}$	$\hat{m}_{8}$	$\hat{m}_{8}$
29	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$	$\hat{m}_{27}$
30	$\hat{m}_{14}$	$\hat{m}_{14}$	$\hat{m}_{14}$	$\hat{m}_{14}$	$\hat{m}_{14}$	$\hat{m}_{14}$
31	$\hat{m}_{10}$	$\hat{m}_{10}$	$\hat{m}_{10}$	$\hat{m}_{38}$	$\hat{m}_{38}$	$\hat{m}_{38}$
32	$\hat{m}_{38}$	$\hat{m}_{38}$	$\hat{m}_{38}$	$\hat{m}_{10}$	$\hat{m}_{10}$	$\hat{m}_{10}$
33	$\hat{m}_{7}$	$\hat{m}_{7}$	$\hat{m}_{7}$	$\hat{m}_{7}$	$\hat{m}_{7}$	$\hat{m}_{7}$
34	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{40}$	$\hat{m}_{4}$
35	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{51}$	$\hat{m}_{40}$
36	$\hat{m}_{4}$	$\hat{m}_{4}$	$\hat{m}_{4}$	$\hat{m}_{4}$	$\hat{m}_{4}$	$\hat{m}_{51}$
37	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{42}$	$\hat{m}_{15}$

38	$\hat{m}_{9}$	$\hat{m}_{15}$	$\hat{m}_{15}$	$\hat{m}_{15}$	$\hat{m}_{15}$	$\hat{m}_{42}$
39	$\hat{m}_{15}$	$\hat{m}_{9}$	$\hat{m}_{9}$	$\hat{m}_{1}$	$\hat{m}_{1}$	$\hat{m}_{1}$
40	$\hat{m}_{1}$	$\hat{m}_{1}$	$\hat{m}_{1}$	$\hat{m}_{20}$	$\hat{m}_{20}$	$\hat{m}_{20}$
41	$\hat{m}_{20}$	$\hat{m}_{20}$	$\hat{m}_{20}$	$\hat{m}_{21}$	$\hat{m}_{21}$	$\hat{m}_{21}$
42	$\hat{m}_{21}$	$\hat{m}_{21}$	$\hat{m}_{21}$	$\hat{m}_{18}$	$\hat{m}_{18}$	$\hat{m}_{18}$
43	$\hat{m}_{18}$	$\hat{m}_{18}$	$\hat{m}_{18}$	$\hat{m}_{9}$	$\hat{m}_{9}$	$\hat{m}_{9}$
44	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{35}$	$\hat{m}_{35}$
45	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$	$\hat{m}_{46}$
46	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$	$\hat{m}_{47}$
47	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$	$\hat{m}_{49}$
48	$\hat{m}_{6}$	$\hat{m}_{6}$	$\hat{m}_{6}$	$\hat{m}_{6}$	$\hat{m}_{6}$	$\hat{m}_{6}$
49	$\hat{m}_{45}$	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{39}$	$\hat{m}_{39}$
50	$\hat{m}_{39}$	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$	$\hat{m}_{45}$
51	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$	$\hat{m}_{13}$
52	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$	$\hat{m}_{11}$

The table, which presents a comparison of Mean Square Error (MSE) and Relative Efficiency (RE), clearly demonstrates the superior performance of the recently developed memory-based ratio estimator $\hat{m}_{52}$ by Noor-ul-Amin in 2021. This estimator consistently outperforms the others in terms of both MSE and RE across various sample sizes. Following closely as the second most efficient estimator across all sample sizes is $\hat{m}_{12}$, which was originally introduced by Quenouille in 1956. It is noteworthy that $\hat{m}_{12}$ maintains a consistently high level of efficiency, demonstrating its robustness and reliability across different sample sizes within our study. These findings underscore the effectiveness and reliability of these two estimators, with $\hat{m}_{52}$, emerging as a particularly noteworthy advancement in the field, showcasing its potential for improved accuracy and efficiency in various statistical applications.

This estimator was modified by Durban (1959). In this estimator author partitioned the data into two equal parts and make use of mean of both parts in the estimators for study and auxiliary variable. The third most efficient estimator for this particular scenario is $\hat{m}_{36}$, which was developed by Subrammani and Prabavathy in 2014. In this estimator, the author incorporates the median of the study and an auxiliary variable. It's worth noting that all three of these estimators are considered efficient for all sample sizes in the study.

Naeem Shahzad and Abida (2022) use simple random sampling to collect data from a population with two characteristics, x and y, where the mean of y is greater than its variance. They compare various existing ratio estimators and propose 52 new ratio estimators based on different approaches. They conduct a simulation study using different scenarios and evaluate the mean square error (MSE) of the estimators to identify the best ones. In order to enhance the performance of some estimators, they additionally employ the correlation coefficient and regression-cum-ratio estimator techniques.

In the ranking, the fourth most efficient ratio estimator is $\hat{m}_{37}$, which was developed by Chakrabarty in 1979. This estimator combines elements of the classical ratio estimator with a simple mean approach. It is noteworthy that $\hat{m}_{37}$, is considered efficient for all sample sizes. The fifth-ranking estimator is $\hat{m}_{17}$, which also proves to be efficient for all sample sizes. This estimator was developed by Kadilar and Cingi (2006), which is the combination of his already proposed estimators by Kadilar and Cingi (2004) and given in $\hat{m}_{6}$ and $\hat{m}_{7}$.

Estimators $\hat{m}_{16}$, $\hat{m}_{19}$, $\hat{m}_{31}$, $\hat{m}_{34}$, $\hat{m}_{43}$, $\hat{m}_{44}$ and $\hat{m}_{50}$ estimators also comes in the orbit of top most 10 ranking of efficient estimators for case two. These estimators were derived by Chakrabarty (1979), Yan and Tian (2010), Subramani and Kumarpandiyan (2012), Jerajuddin and Kishun (2016), Abdullahi and Yahaya (2017) , Tailor and Sharma (2009) and Kadilar and Cingi (2006) respectively. The use of these estimators, which incorporate sample size, coefficient of variation, kurtosis measure, and correlation coefficient of auxiliary variables, suggests that incorporating these measures in the construction process can be beneficial across all sample sizes.

The ratio estimator with the poorest performance is referred to as $\hat{m}_{11}$, which was developed by Goodman and Hartley in 1958 and this estimator incorporates sample size, population size, and the correlation coefficient in its construction. Unfortunately, $\hat{m}_{11}$ exhibits the worst performance across all sample sizes. The second poorest performing estimator, in terms of mean squared error (MSE) and relative efficiency (RE), is $\hat{m}_{13}$, which was introduced by Kadilar and Cingi in 2003. $\hat{m}_{13}$ relies on the square of the mean of an auxiliary variable for both sample and population data. Regrettably, the performance of $\hat{m}_{13}$ also ranks poorly for all sample sizes.

CONCLUSION

In this paper, we propose a new ratio estimator that is more efficient than the conventional ratio estimator when the population variance is less than the population mean in simple random sampling. The proposed estimator is based on the ratio of the sample mean and the sample median of the variable of interest, and it incorporates the information on the population size and the sample size. We derive the bias and the mean squared error of the proposed estimator, and compare its performance with some existing ratio estimators using simulated and real data sets. The results show that the proposed estimator has smaller bias and mean squared error than the conventional ratio estimator and some other ratio estimators under certain conditions. The study identifies several ratio estimators, including Estimators $\hat{m}_{16}$, $\hat{m}_{19}$, $\hat{m}_{31}$, $\hat{m}_{34}$, $\hat{m}_{43}$, $\hat{m}_{44}$ and $\hat{m}_{50}$which consistently rank among the top 10 most efficient estimators across different sample sizes and scenarios. These estimators take into account critical factors such as sample size, coefficient of variation, kurtosis measure, and the correlation coefficient of auxiliary variables. Their incorporation in the estimation process proves beneficial across a wide range of sample sizes. On the other hand, $\hat{m}_{11}$developed by Goodman and Hartley in 1958, emerges as the poorest performing estimator across all sample sizes. This estimator incorporates sample size, population size, and the correlation coefficient in its construction. $\hat{m}_{13}$ introduced by Kadilar and Cingi in 2003, ranks as the second poorest performing estimator, with suboptimal performance in terms of Mean Square Error (MSE) and Relative Efficiency (RE) across various sample sizes.

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